بسم الله الرحمن الرحيم

Sudan University of Science and Technology

School of Electronics Engineering

Fuzzy Control

التحكم الغامض

Lecture (1)

Instructor: Dr. Mohammed Elnour A/Alla

Course Outlines

- Introduction
- Fuzzy Set Theory
- Fuzzy Relations
- Fuzzy Rules and Fuzzy Reasoning
- Fuzzy Inference systems
- Introduction to MATLAB Fuzzy logic tool box
- Fuzzy decision making

References

1. Leonid Reznik " Fuzzy controllers", ebook
2. Jan Jantzen ,''Foundations of Fuzzy Control'', 2007 John Wiley & Sons Ltd, ebook
3. Kevin M. Passino,"Fuzzy Control ", ebook
4. S. N. Sivanandam, et, al ''Introduction to Fuzzy Logic using MATLAB" ebook (numerical Exs)
5. Fuzzy Logic Toolbox. For use with Matlb. Users guide. Ver. 2

…..References

6. J. S. R. Jang “Neuro Fuzzy and Soft Computing. A comprehensive approach to Learning and Machine Intelligence
7. Fuzzy sets and system; Theory and applications ebook
8. Guanrong Chen (Introduction to Fuzzy Sets, Fuzzy Logic and Fuzzy Control Systems) ebook
9. S. Sumathi and Surekha P. (Computational Intelligence Paradigms :Theory and Application using matlab)

INTRODUCTION

- *I understand that you want to start a design immediately.*
- *As you know nothing about fuzzy control, we will base our design just on our human experience.*
- *Let us consider a classical control problem. We want to control the boat movement which we should drive along the straight line from point A to point B.*
- ***How will you design a controller to do it?***

Introduction …

- *Well, firstly I must derive a mathematical model of the plant and then develop a mathematical model of a controller*.

- And how will you develop the controller model?

- *I will apply one of the design methods, e.g., pole placement design, and obtain a transfer function for the controller.*

Introduction …

- *OK. Now suppose we do not know exactly the mathematical model of our plant. Moreover, we do not know any classical design method. What can we do then?*
- *In other words, is it possible to do control without the mathematical model?*
- *Actually in our daily life we do many control activities without solving differential equations. e.g: ride bicycles, park cars etc..*

Introduction …

- *Try to use your own experience. How did you drive a boat in your childhood?*
- *I turned the rudder left or right depending on the position of a boat.*
- *Great! So we can say that:*
  - *if the boat is situated exactly on the line we should not do anything,*
  - *if the boat is situated to the left of the line we should turn the rudder to the right (let us call this direction positive),*
  - *and if the boat is positioned to the right of the line we should turn the rudder to the left.*

Introduction …

- Now let us try to formulate this control law as a set of rules.
- **If** deviation is *zero* **then** turn is *zero*.
- **If** deviation is *positive* **then** turn is *negative.*
- **If** deviation is *negative* **then** turn is *positive*.

Rule base

Introduction …

- *Are you sure this table can control the boat movement?*
- *Of course, very roughly.*
- *To improve the control quality and make our controller more reactive, we need to increase the number of values describing each variable.*
- *Now let us use **small**, **medium** and **big** values for both the **deviation** and the **turn**. Then we obtain the rules table:*

Linguistic variables

Linguistic values

Deviation	NB	NM	NS	Z	PS	PM	PB
Turn	PB	PM	PS	Z	NS	NM	NB

NB: Negative Big, NM: Negative Medium, NS: Negative Small
Z: Zero, PS: Positive Small, PM: Positive Medium

Introduction …

What is Fuzzy Control?

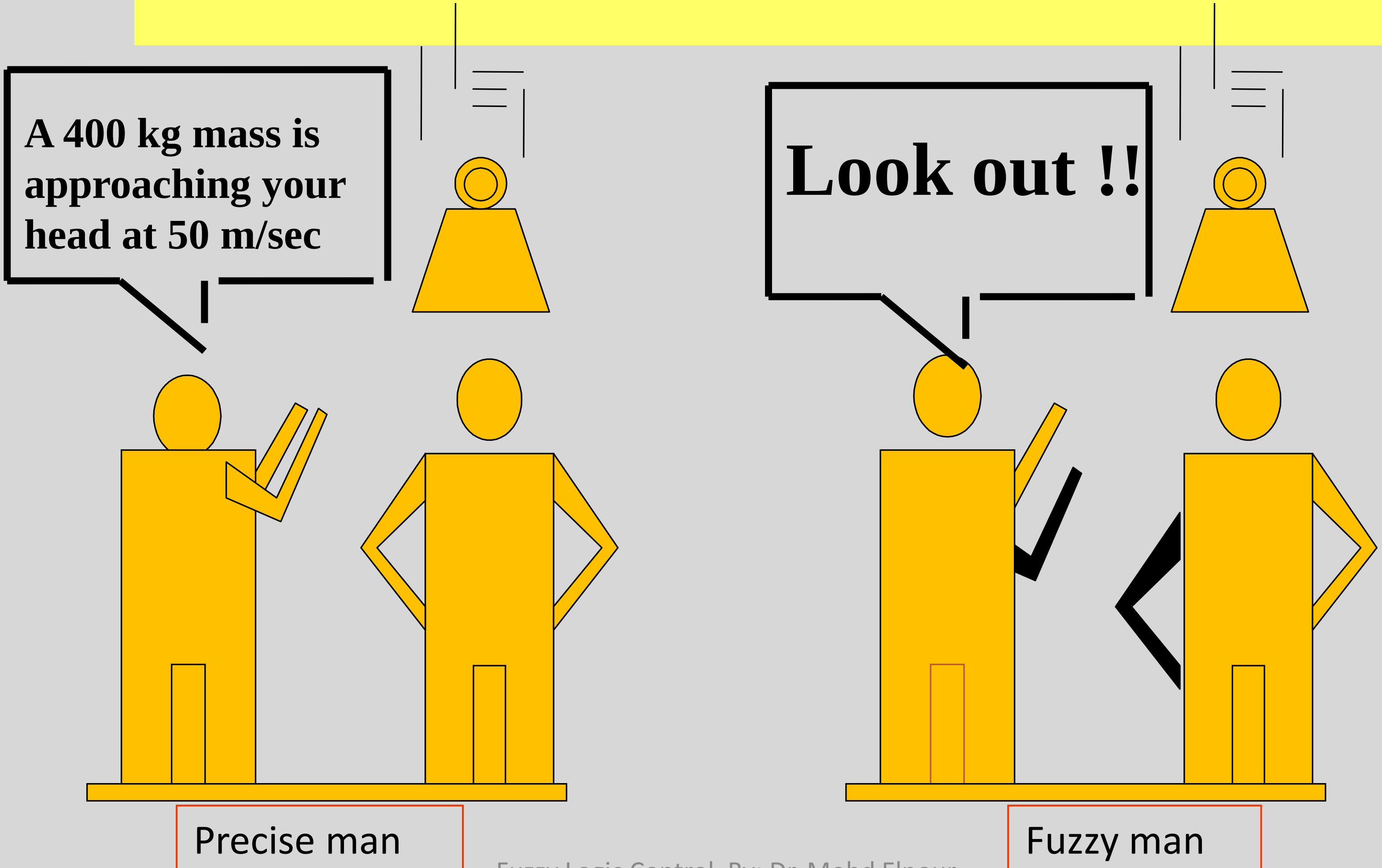
Fuzzy control is one of the *intelligent control* techniques that uses *fuzzy logic* to model the control strategies of an expert operator, which are described in linguistic fuzzy (imprecise) terms .

What Is Fuzzy Logic?

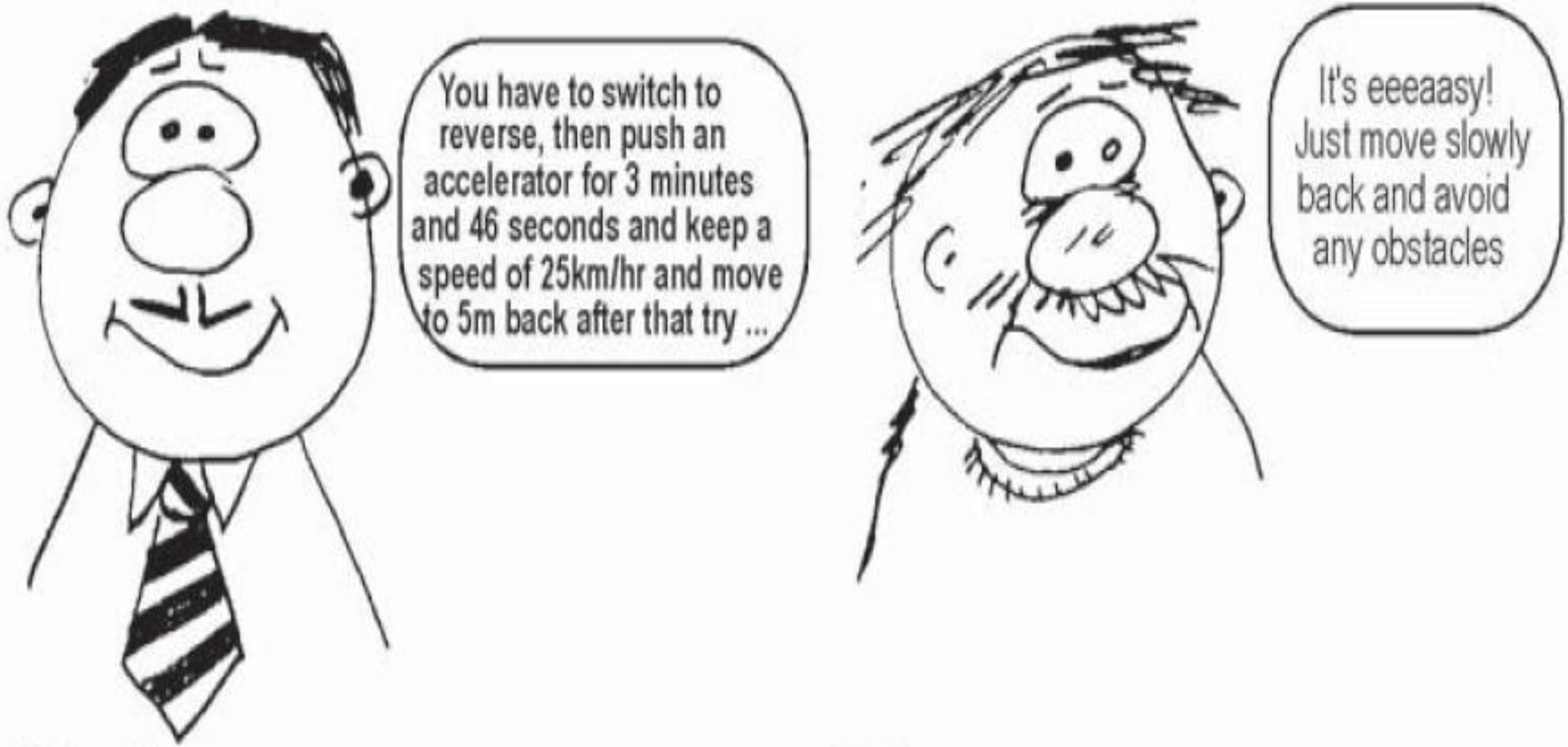
- *Humans are more efficient in dealing with fuzzy data than computers (e.g crossing a busy road).*
- *Fuzzy logic is used to convey the human capability of handling fuzzy information to the computer*
- *It is a mathematical tool to capture and handle the fuzzy data that is used in the natural language*

What Is Fuzzy Logic?

Significance of Precision in the Real World



How are you going to park a car?



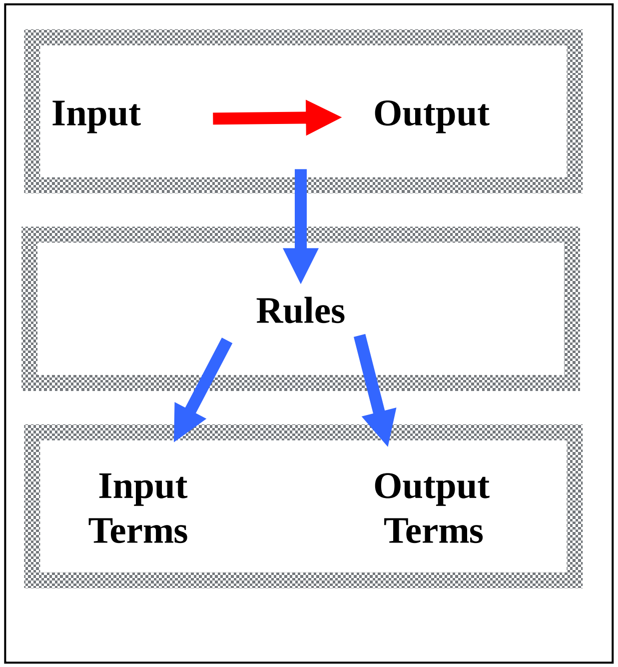
Crisp man **Fuzzy man**

What Is Fuzzy Logic?

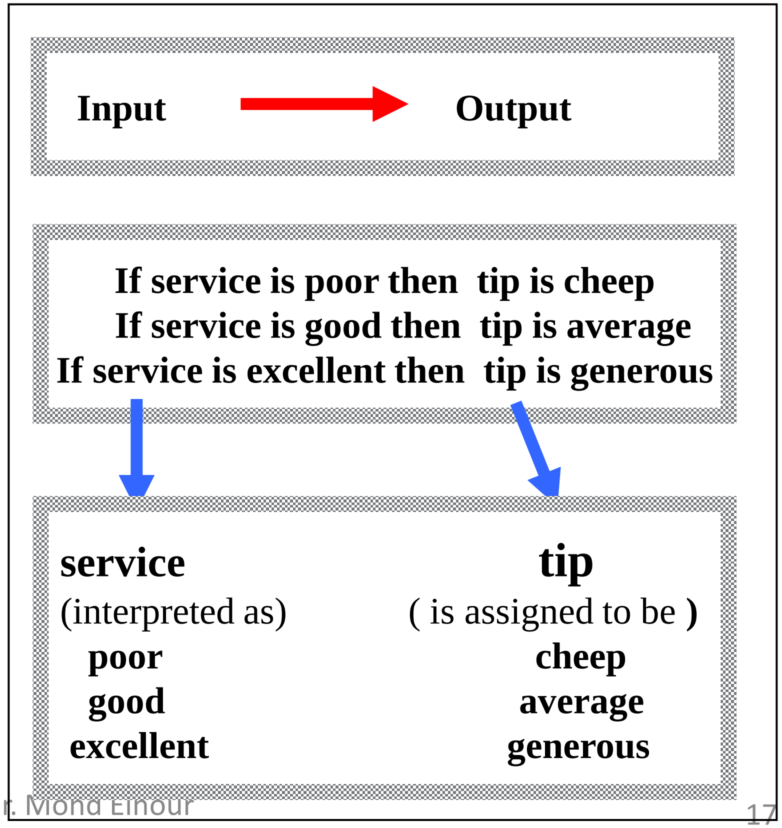
-The Big Picture-

FL maps an input space to an output space using a list of if-then rules

The General Case



A Specific Example



What Is Fuzzy Logic?

-Linguistic Rules,Variables and values

Rules

If service is poor then tip is cheep

If service is good then tip is average

If service is excellent then tip is generous

Input Linguistic variable

Linguistic Values

Output Linguistic variable

Linguistic Values

Linguistic Rules of a fuzzy systems are the laws it execute

Linguistic variables **defines a concept of our everyday language**

Linguistic values describe the characteristic of the *Linguistic variables*

Why fuzzy logic?

Fuzzy Logic can:

- ❑ *Represent vague language naturally*
- ❑ *Enrich not replace crisp sets*
- ❑ *Allow flexible engineering design*
- ❑ *Improve model performance*
- ❑ *Are simple to implement*
- ❑ *And best of all, they often work!*

Why do we need this new theory?

Limitations of conventional controllers

● Plant nonlinearity. The efficient linear models of the process are too restrictive. Nonlinear models are computationally intensive and have complex stability problems.

● Plant uncertainty. A plant does not have accurate models due to uncertainty and lack of perfect knowledge.

● Uncertainty in measurements. Uncertain measurements do not necessarily have stochastic noise models.

● Temporal behaviour. Plants, controllers, environments and their constraints vary with time. Moreover, time delays are difficult to model.

Why do we need this new theory?

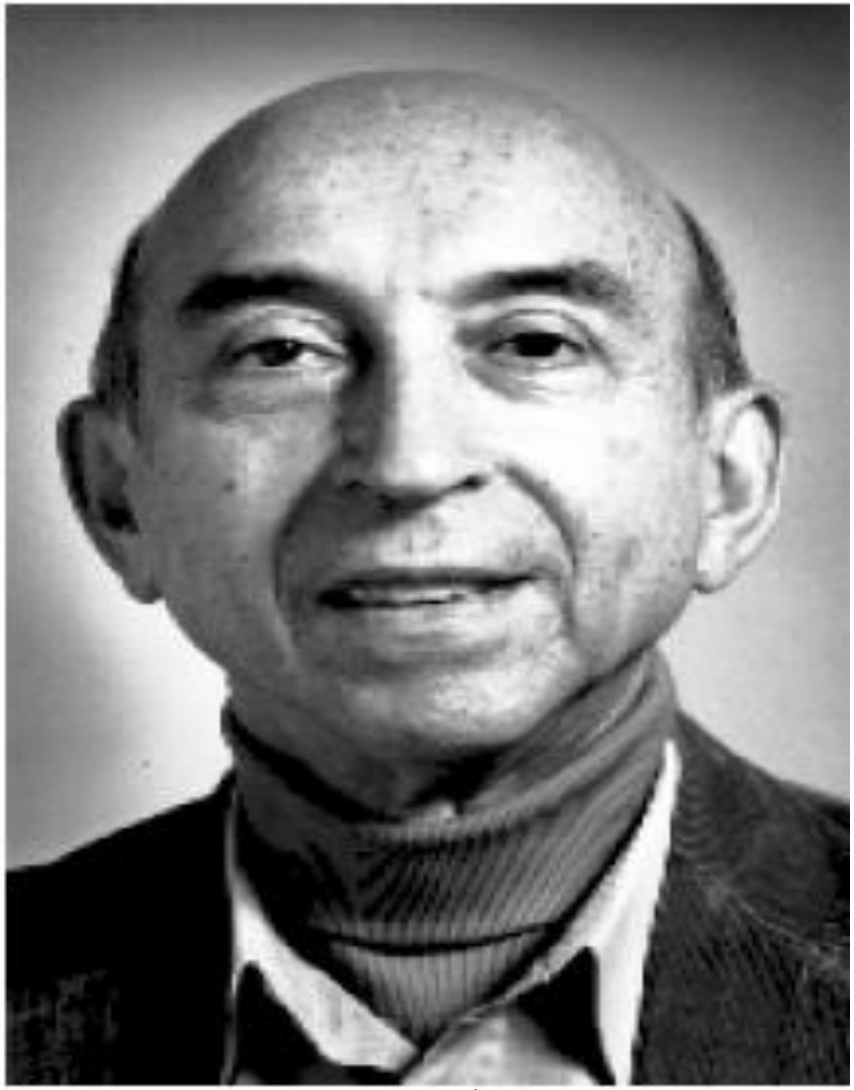
Benefits of the fuzzy controllers

- *Fuzzy controllers are more robust than PID controllers because they can cover a much wider range of operating conditions than PID can, and can operate with noise and disturbances of different natures.*
- *Developing a fuzzy controller is cheaper than developing a model-based or other controller to do the same thing.*
- *Fuzzy controllers are customisable, since it is easier to understand and modify their rules, which not only use a human operator's strategy but also are expressed in natural linguistic terms.*
- *It is easy to learn how fuzzy controllers operate and how to design and apply them to an application*

The root and development of fuzzy logic

- *Fuzzy set thory was introduced by Professor Lotfi Zadeh (USA) in 1965. as an extension of the classical set thory*
- *1972 First working group on fuzzy systems in Japan by Toshiro Terano*
- *1973 Paper about fuzzy algorithms by Zadeh (USA)*
- *1974 Steam engine control by Ebrahim Mamdani (UK)*
- *Too many events, inventions and projects to mention till 1991*
- *After 1991 fuzzy technology came out of scientific laboratories and became an industrial tool.*
- *In the last two decades, the fuzzy sets theory has established itself as a new methodology for dealing with any sort of ambiguity and uncertainty.*

Zadeh (USA)



What are the main areas of fuzzy logic applications?

The following list includes just a small number of successful projects and is intended to demonstrate a huge diversity of possible applications.

- Automatic control of dam gates for hydroelectric power plants (Tokyo Electric Power.)
- Simplified control of robots (Hirota, Fuji Electric, Toshiba, Omron)
- Camera-aiming for the telecast of sporting events (Omron)
- Efficient and stable control of car engines (Nissan)
- Cruise-control for automobiles (Nissan, Subaru)
- Automatic transmission of automobiles (see next slide)
- Substitution of an expert for the assessment of stock exchange activities (Yamaichi, Hitachi)
- Optimised planning of bus timetables (Toshiba, Nippon-System, Keihan-Express)
- Archiving system for documents (Mitsubishi Elec.)
- Prediction system for early recognition of earthquakes (Seismology Bureau of Metrology, Japan)

1996 Saturn SL1 and Mitsubishi Galant ES and LS are equipped with fuzzy controlled automatic transmission

Applications cont.

❖Medicine technology: cancer diagnosis (Kawasaki Medical School)

❖ Automatic motor-control for vacuum cleaners with a recognition of a surface condition and a degree of soiling (Matsushita)

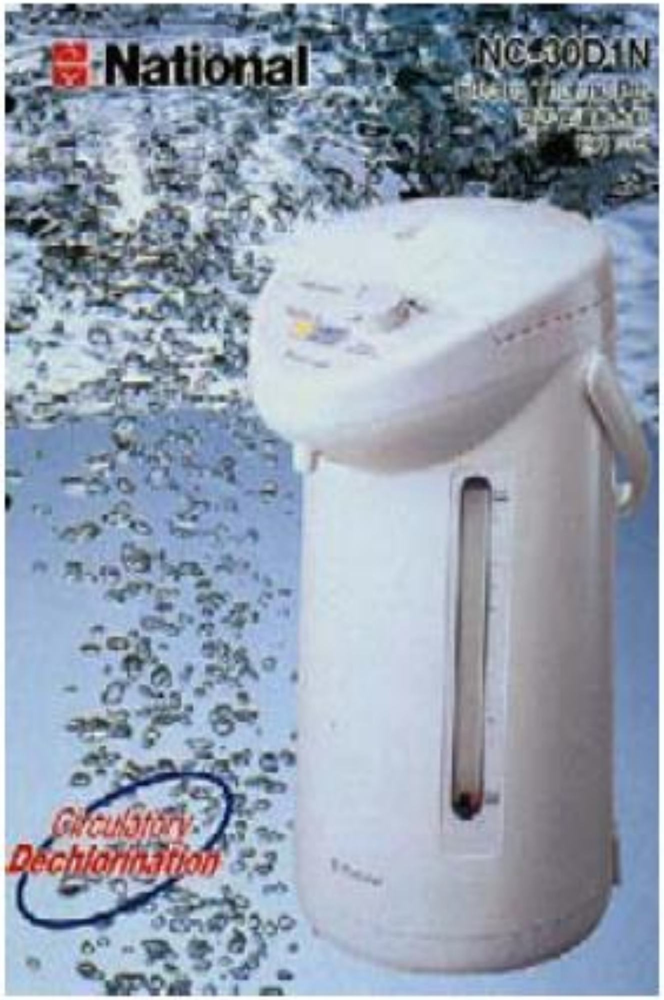
During the ‘fuzzy’ revolution, the fuzzy technology was introduced not only to a world of complex industrial projects, but to simple everyday home appliances: next are some examples

Panasonic®/National® Fuzzy Logic

Rice cooker

Fuzzy logic controls the cooking process, self adjusting for rice and water conditions

National® Deluxe Electric Fuzzy Logic



Thermo pot

This unit represents the best technology available in producing clean boiled water on demand for making tea. It is fuzzy logic computer controlled

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Lecture (2)

FUZZY SETS

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Fuzzy Sets: Outline

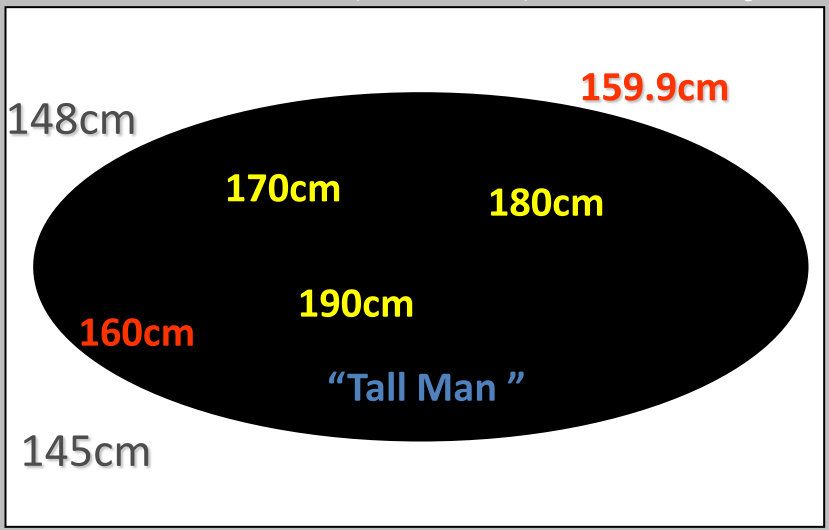
- Introduction
- Basic definitions and terminology
- MF formulation and parameterization
- Set-theoretic operations
  - Union
  - Intersection
  - complement
- Properties of fuzzy sets

Fuzzy Sets

- A classical *set X* is a collection of definite, distinguishable objects of our intuition that can be treated as a whole. The objects are the *members* of *X*
- A crisp (classical) set is a set for which each value either is or is not contained in the set.
- For a fuzzy set, every value has a membership value, and so is a member to some extent.
- The ***membership value*** defines the extent to which a variable is a member of a fuzzy set.
- The membership value is from 0 (not at all a member of the set) to 1.

Fuzzy Set

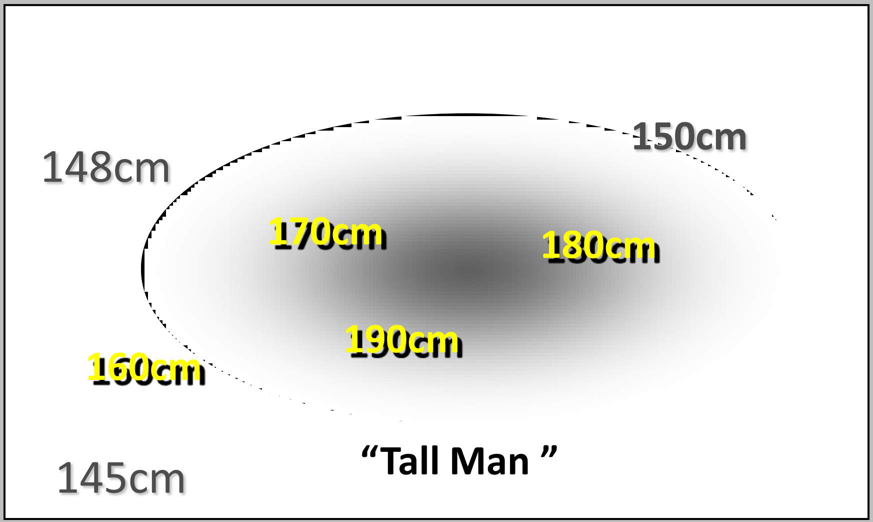
Conventional (Boolean) Set Theory



Container boundaries that include or exclude elements

"More-or-Less" Rather Than "Either-Or" !

Fuzzy Set Theory

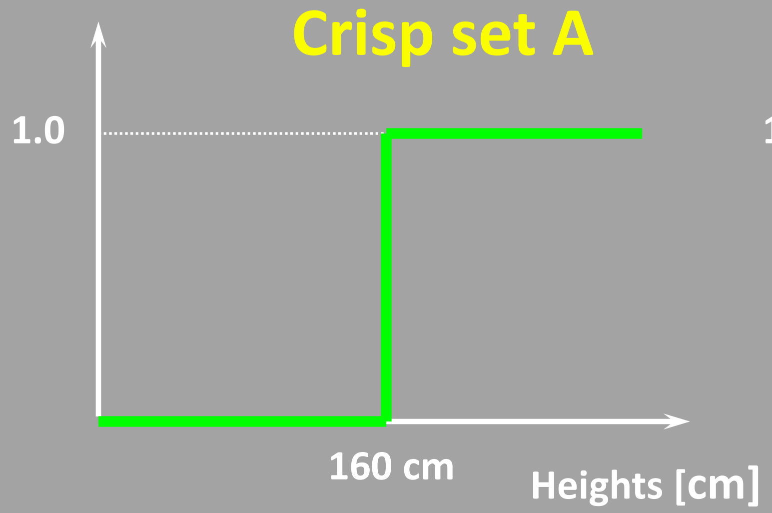


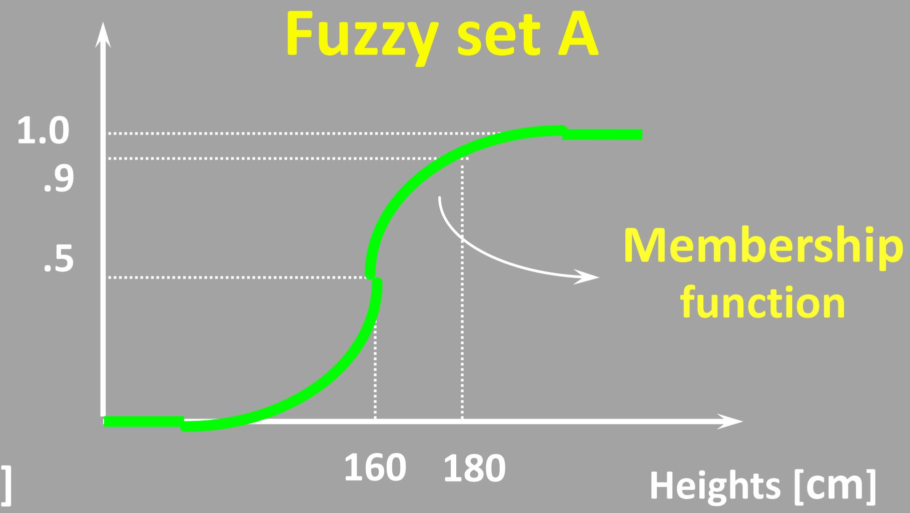
No clearly defined boundaries

Fuzzy Sets

- Sets with fuzzy boundaries

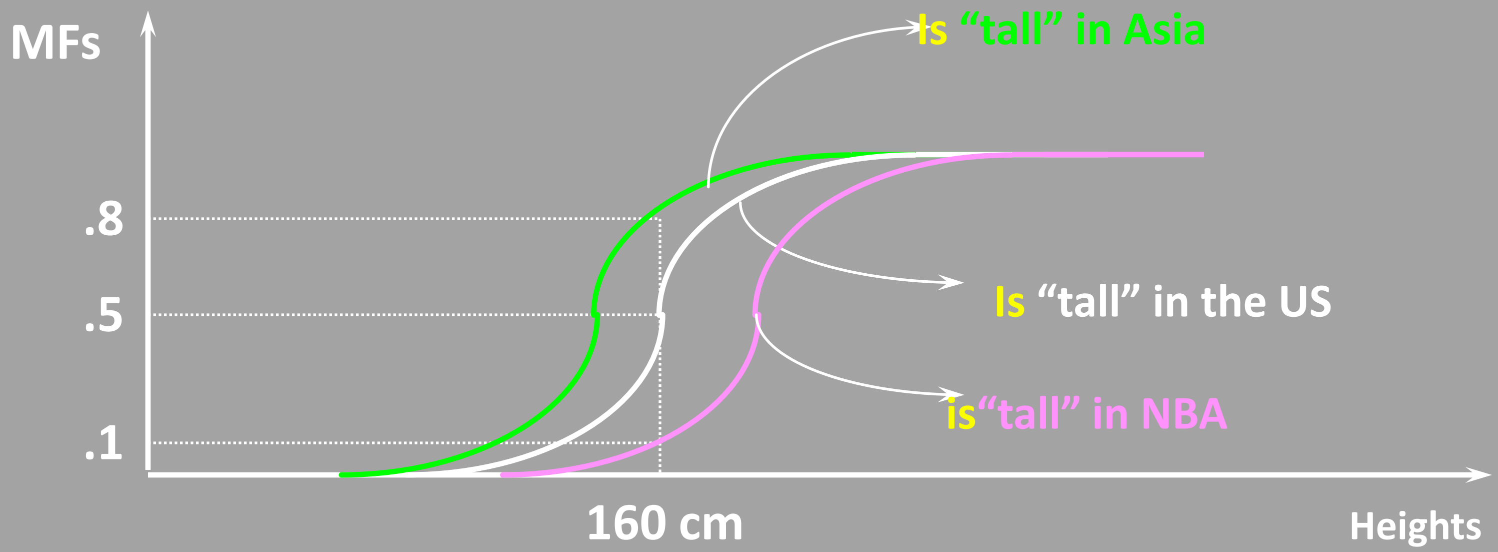
A = Set of tall people





Membership Functions (MFs)

- Characteristics of MFs:
  - Subjective measures
  - Not probability functions



Fuzzy Sets

- Formal definition:

  A fuzzy set *A* in *X* is expressed as a set of ordered pairs:

$$\underbrace{A}_{\text{Fuzzy set}} = \{(x, \underbrace{\mu_A(x)}_{\text{Membership function (MF)}}) \mid x \in \underbrace{X}_{\text{Universe or universe of discourse}}\}$$

A fuzzy set is totally characterized by a membership function (MF).

Fuzzy Sets with Discrete Universes

- Fuzzy set C = “desirable city to live in”

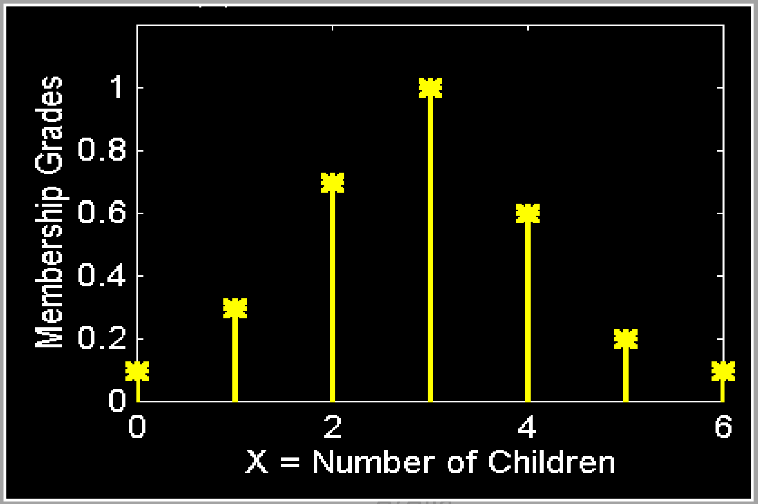
  X = {Khartoum, Cairo, Meca,} discrete and nonordered UOD

  C = {(Me, 0.9), (Kh, 0.8), (Ca, 0.6)}

- Fuzzy set A = “preferred number of children”

  X = {0, 1, 2, 3, 4, 5, 6} (discrete ordered UOD)

  A = {(0, .1), (1, .3), (2, .7), (3, 1), (4, .6), (5, .2), (6, .1)}



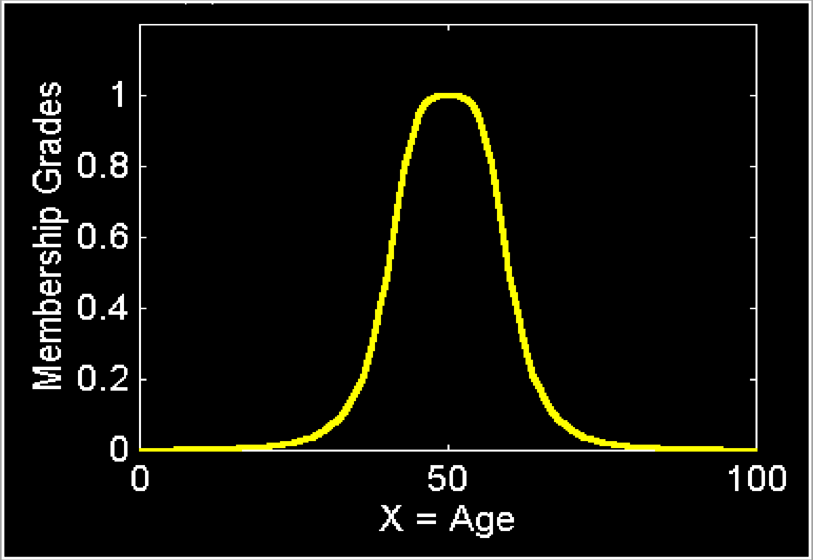
Fuzzy Sets with Cont. Universes

- Fuzzy set B = "about 50 years old"

  X = Set of positive real numbers (continuous)

  B = {(x, μ_B(x)) | x in X}

$$\mu_B(x) = \frac{1}{1 + \left(\frac{x-50}{10}\right)^2}$$



Alternative Notation

- A fuzzy set A can be alternatively denoted as follows:

X is discrete $\Rightarrow$ $$A = \sum_{x_i \in X} \mu_A(x_i) / x_i$$

X is continuous $\Rightarrow$ $$A = \int_X \mu_A(x) / x$$

Note that Σ and integral signs stand for the union of membership grades; "/" stands for a marker and does not imply division.

Alternative Notation Example

- The fuzzy set of C in the previous example can be expressed as:

C=0.9/Me+0.8/Kh+0.6/Ca

the set A as:

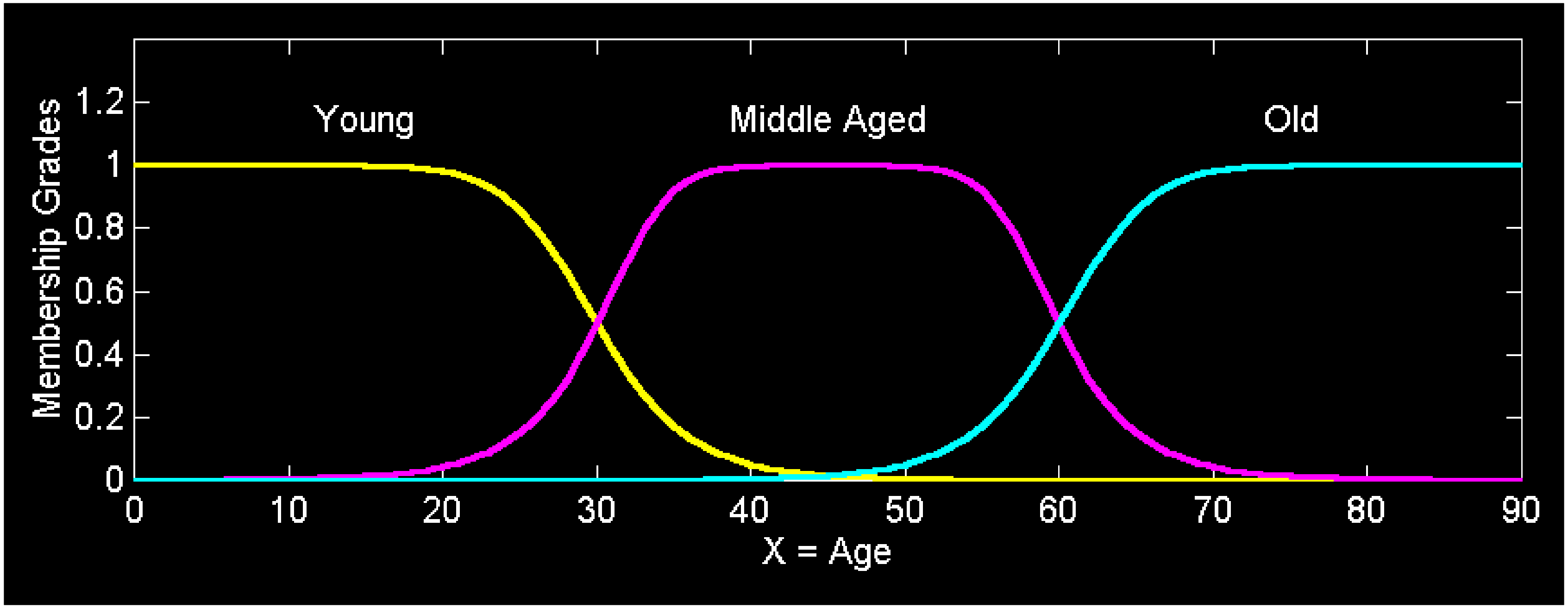
A=0.1/0+0.3/1+0.7/2+1/3+0.7/4+0.3/5+0.1/6

And the set B as:

$$B = \int_{R^+} \frac{1}{1+\left(\frac{x-50}{10}\right)^2} \Big/ x$$

Fuzzy Partition

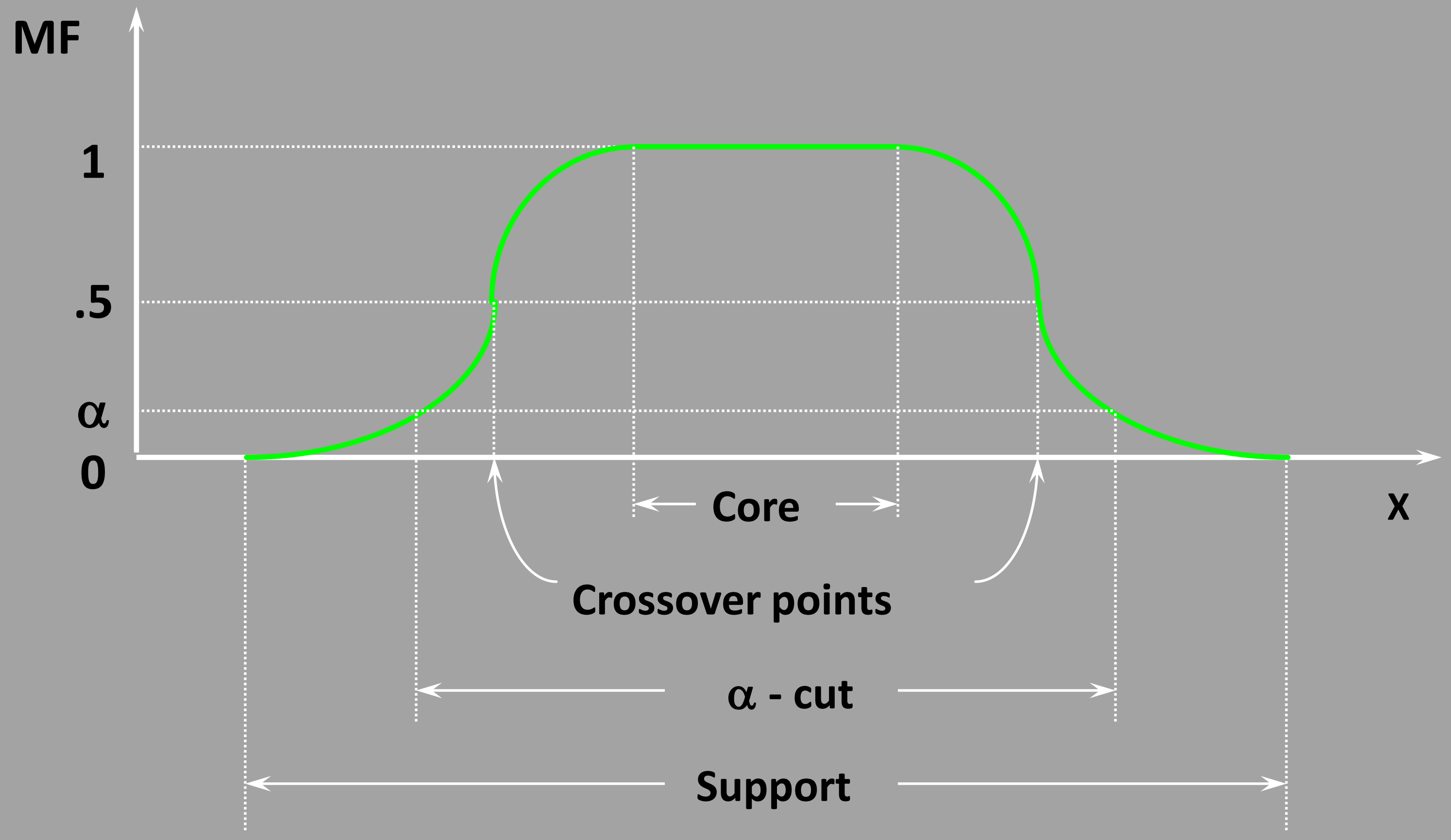
- In practice when the UOD X is continuous, we usually partition it into several fuzzy sets
- Fuzzy sets used for partitioning are called linguistic values, e.g. {“young”, “middle aged”, and “old”}:

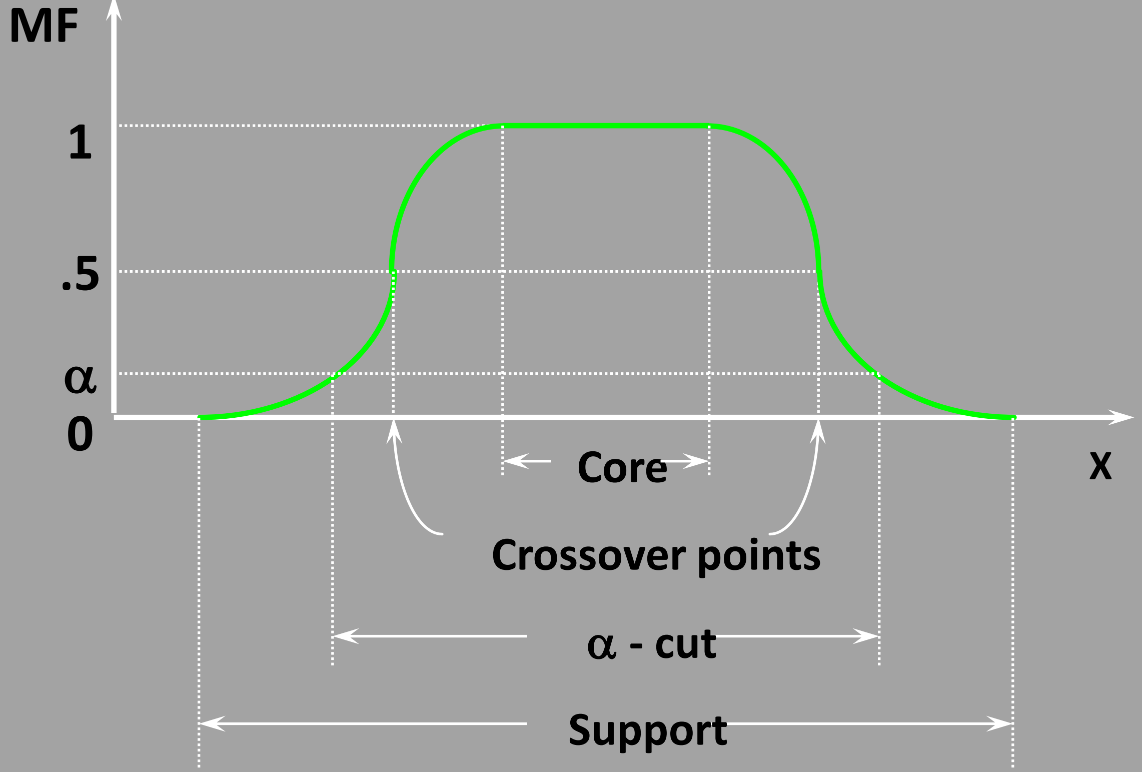


More Definitions

- Support
- Core
- Normality
- Crossover points
- Fuzzy singleton
- α-cut, strong α-cut
- Convexity
- Bandwidth
- Symmetricity

MF Terminology



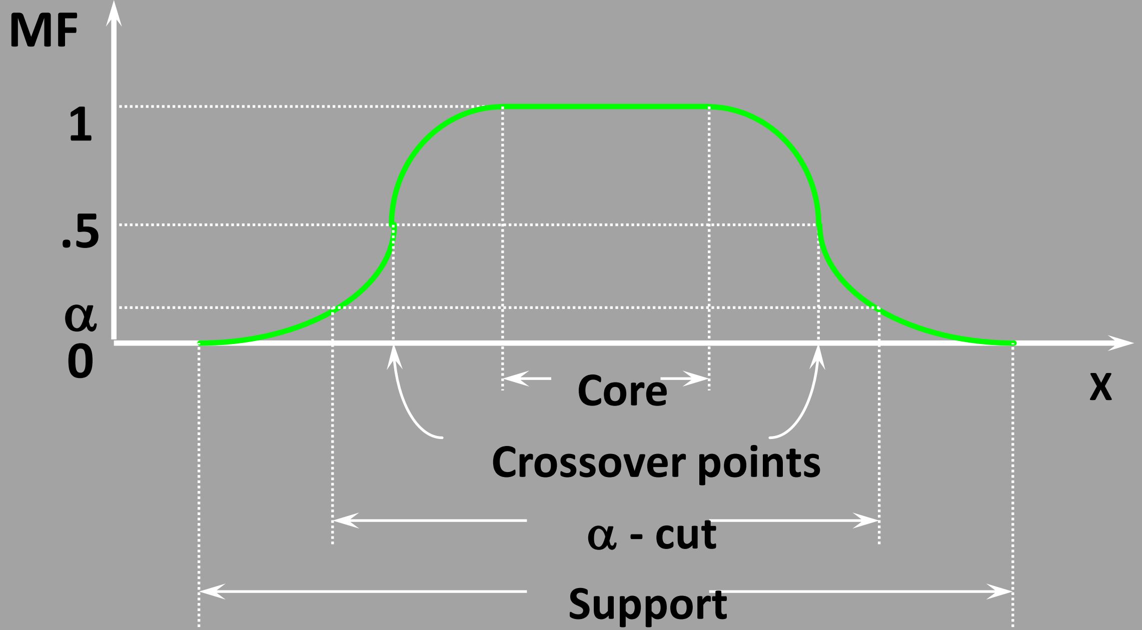


Def: Support

The support S(A) of a fuzzy set A is the crisp set of all the elements of the universal set (UOD) for which membership function has non-zero value

$S(A) = \{ u \in U / \mu_A(u) > 0 \}$

$$\text{Support}\ (A) = \{ x \mid \mu_A(x) > 0 \}$$



Def: α – cut (or α lvel) set

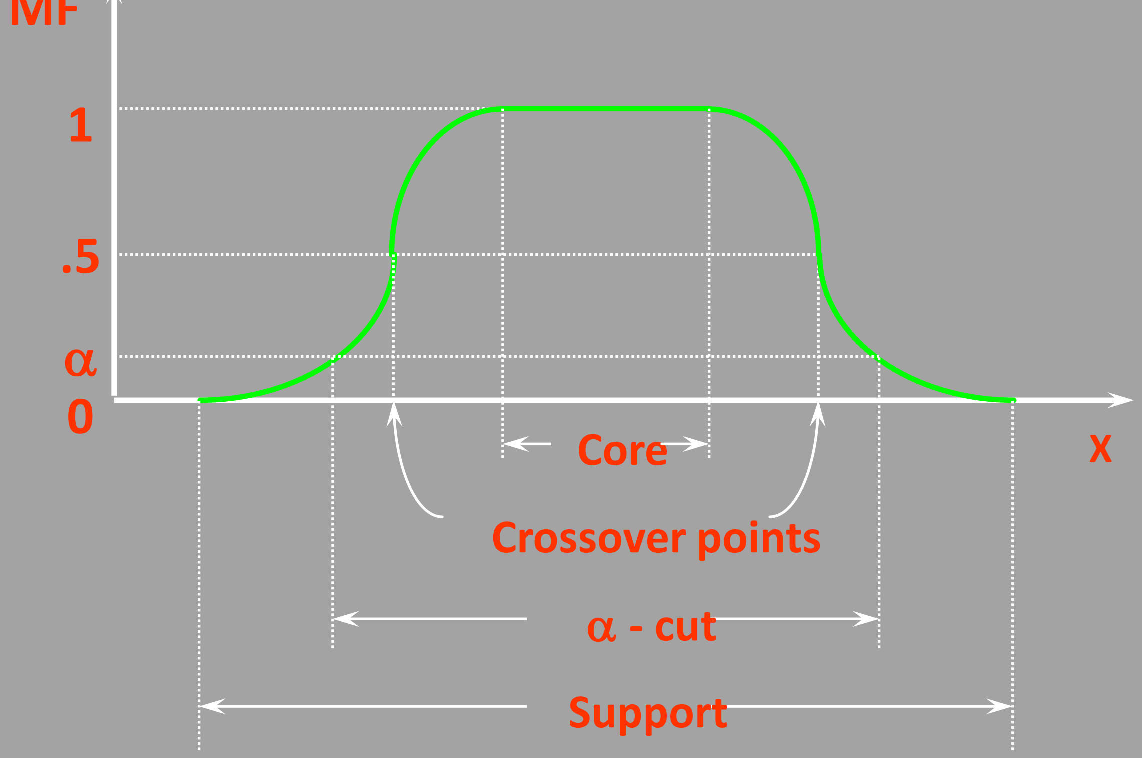
The set of elements that belong to the fuzzy set A at least to the degree α is called the α-level-set or α-cut-set

$$A_\alpha = \left\{ x \middle| \mu_A(x) \geq \alpha \right\}$$

By the way, what do you think, either an α-level-set is crisp or fuzzy?

As it includes just elements without their membership degrees, it is crisp.

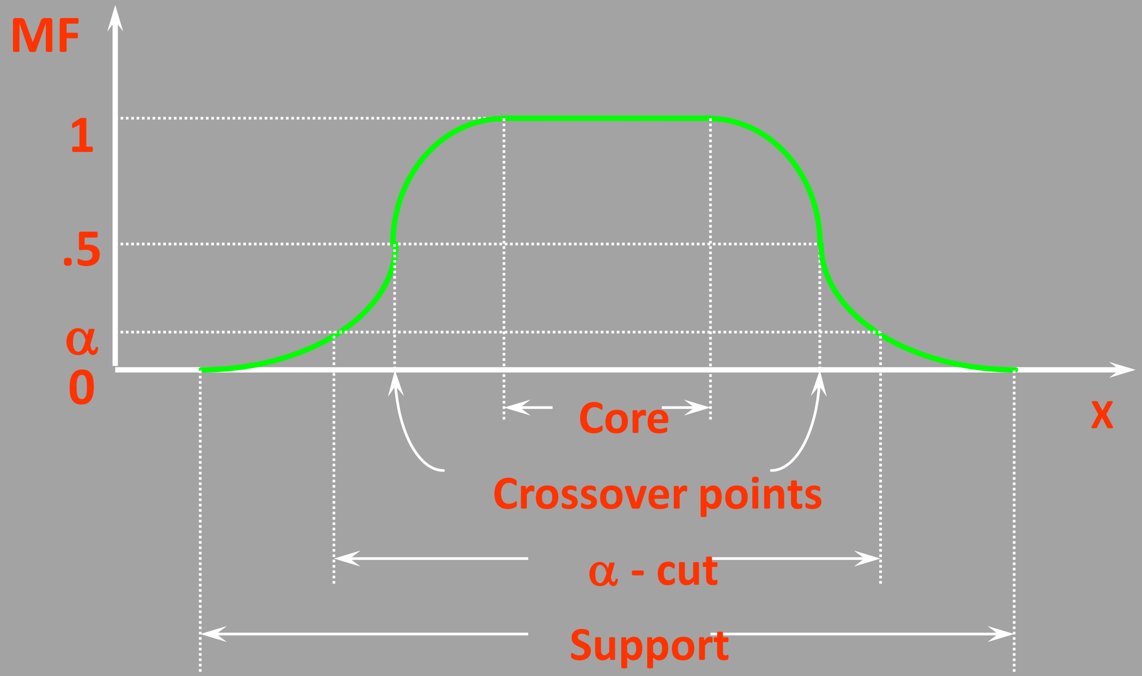
Strong α – cut is defined by $$A'_\alpha = \left\{ x \middle| \mu_A(x) \succ \alpha \right\}$$



Def: Crossover point

The element of the universal set, for which the membership function has the value of 0.5, is called a crossover point.

$$Crossover(A) = \{x | \mu_A(x) = 0.5\}$$



Def: **Core:**

Is the set of all elements x in X that belong to the fuzzy set A such that $\mu_A(x)=1$:

$$Core(A)=\{x \mid \mu_A(x)=1\}$$

MF Terminology cont..

Definition Normality

A fuzzy set is normal if its core is nonempty. In other words, we can always find a point x ε X such that $\mu_A(x)=1$

Definition Height of a fuzzy set

The height of a fuzzy set A, hgt(A) is given by a supremum of the membership function over all u$\in$U

$$\mathbf{hgt(A) = sup_U\ \mu_A(\ u\ )}$$

(Supremum in this definition means the highest possible (or almost possible) degree.)

MF Terminology cont..

Definition Singleton

Is a fuzzy set whose support is a single point in X with $\mu_A(x)=1$

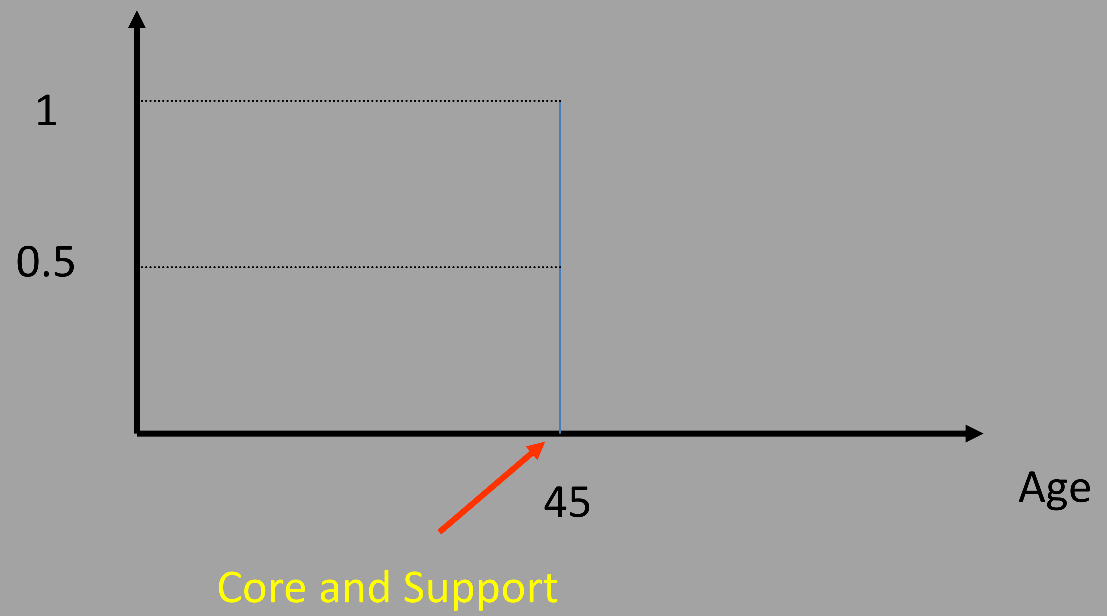


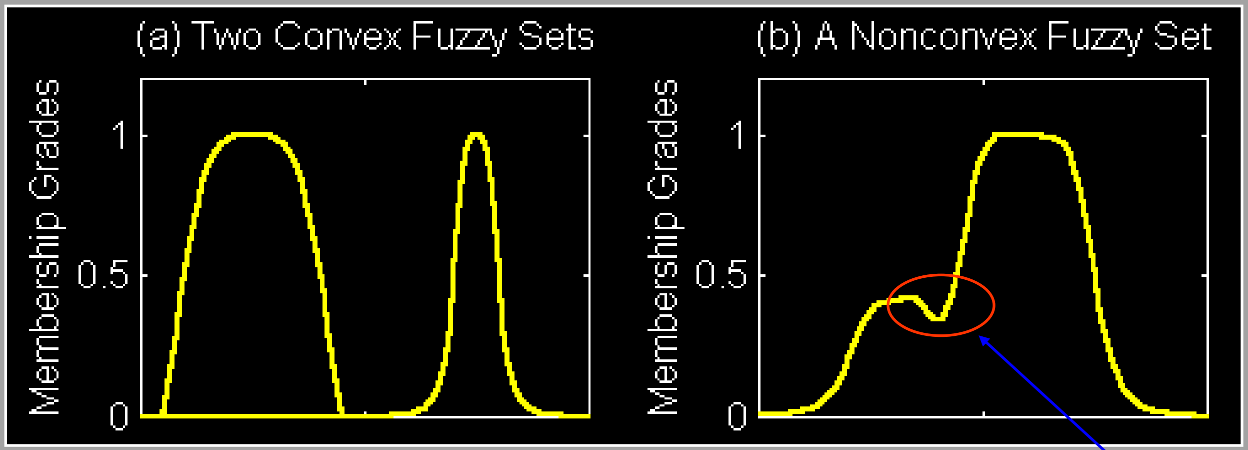
Fig: fuzzy singleton “45 year old”

Convexity of Fuzzy Sets

- A fuzzy set *A* is convex if for any λ in [0, 1],

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

Alternatively, *A* is convex if all its α-cuts are convex.



convexmf.m

Condition is not satisfied

MF Terminology cont..

Definition Bandwidth:

for a Normal and Convex Fuzzy Set the bandwidth or width is the distance between the two unique crossover point

$$width\ \ (A) = \left| x_2 - x_1 \right|,$$

$$where \quad \mu_A(x_1) = \mu_A(x_2) = 0.5$$

MF Terminology cont..

Definition: Symmetry:

A Fuzzy Set is symmetric if its MF is symmetric around a certain point x=c, namely

$$\mu_A(c+x)=\mu_A(c-x)\ for\ \ all\ \ \ x\in X$$

Formulation of Commonly Used MFs

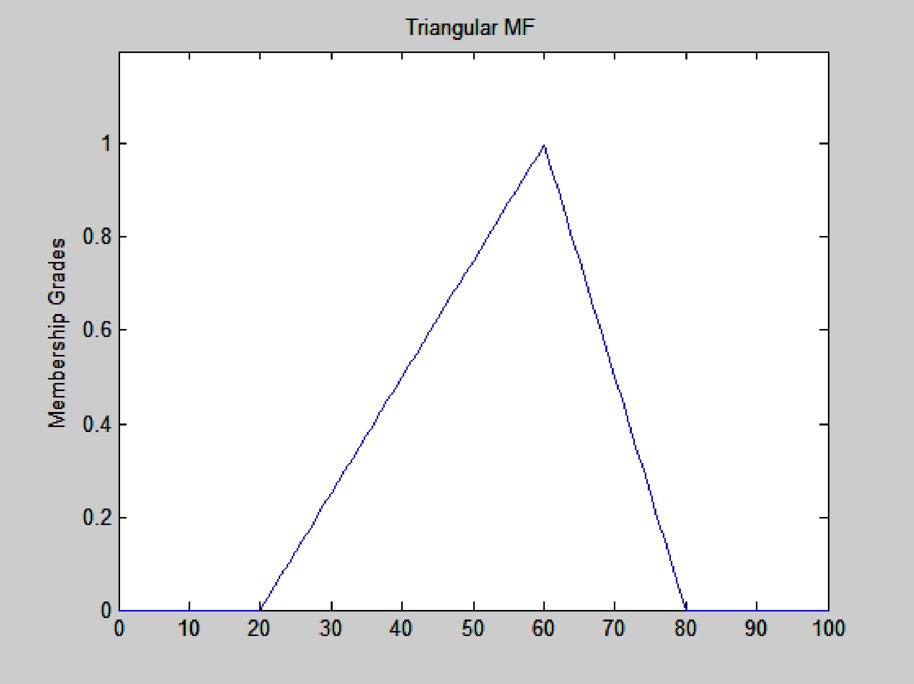
MFs of one dimension

- Triangular MF: is specified by three parameters{ a,b ,c} as follows:

$$triangle(x,a,b,c)=\begin{cases} 0 & x\le a \\ \frac{x-a}{b-a} & a\le x\le b \\ \frac{c-x}{c-b} & b\le x\le c \\ 0 & c\le x \end{cases}$$

Matlab code

```
x = 0:100;
mf = trimf (x, [20, 60, 80]);
plot(x, mf);
axis([-inf inf 0 1.2]);
ylabel('Membership Grades');
 title ('Triangular MF');
```



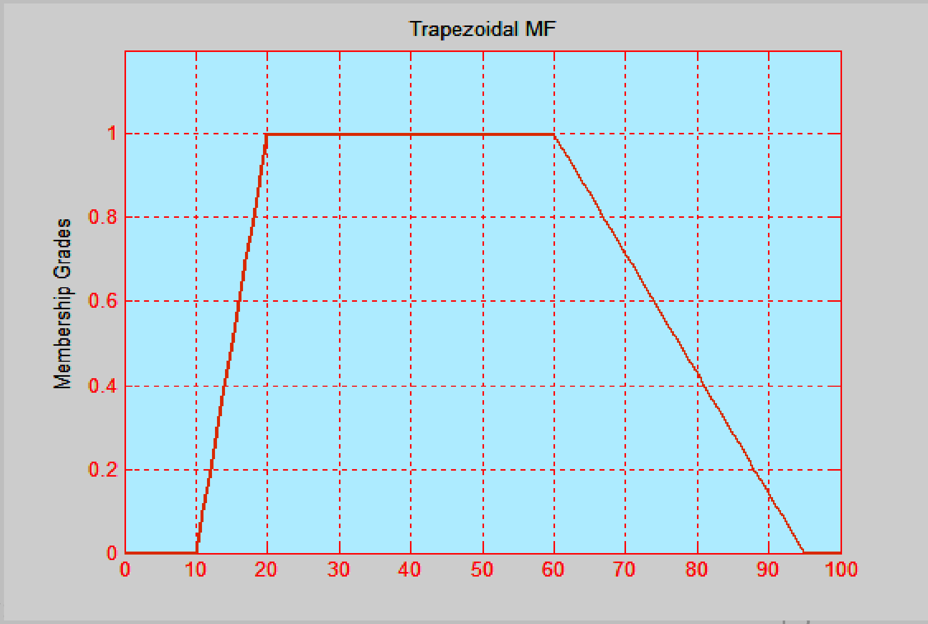
Formulation of Commonly Used MF
MFs of one dimension

- **Trapezoidal MF:** is specified by four parameters{ a,b ,c,d } as follows:

$$trapezoid(x,a,b,c,d) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{d-c} & c \le x \le d \\ 0 & x \ge d \end{cases}$$

Matlab code

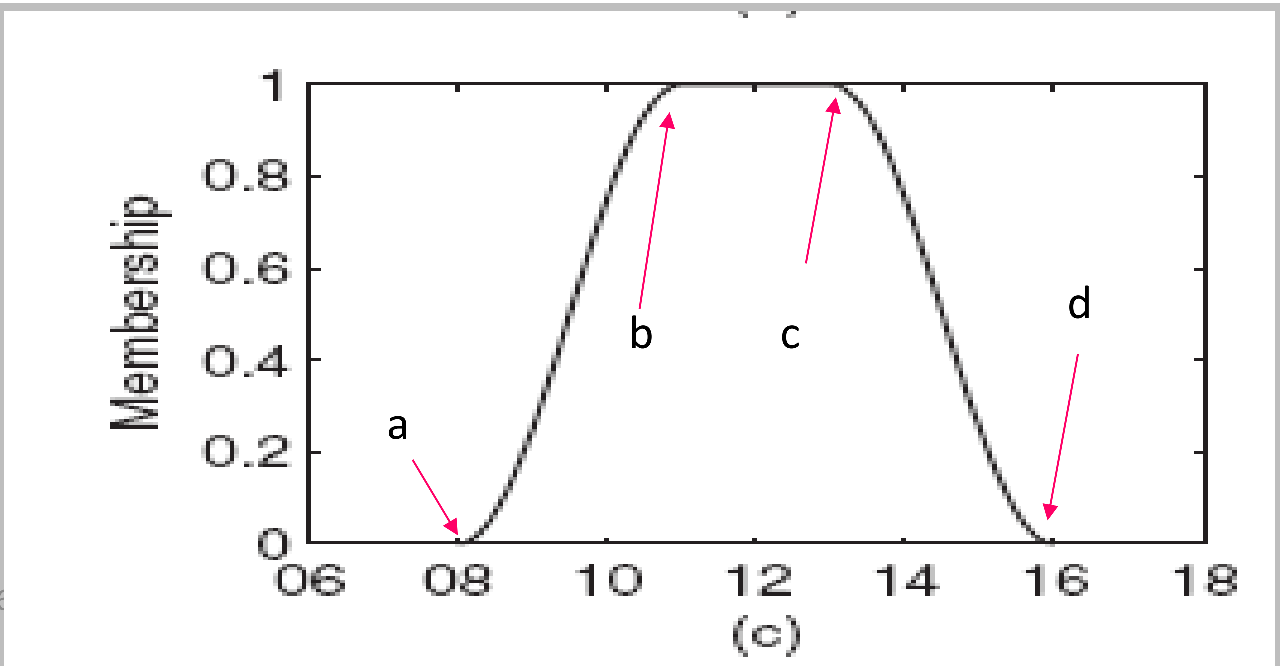
```
x = 0:100;
mf = trapmf(x, [10, 20, 60, 95]);
 plot(x, mf);
axis([-inf inf 0 1.2]);
ylabel('Membership Grades');
title(' Trapezoidal MF');
```



Formulation of Commonly Used MF
MFs of one dimension

- Smooth Trapezoidal MF: is specified by four parameters{ a,b ,c,d } as follows:

$$\mu_{STrapezoid}(x;a,b,c,d)=\left\{\begin{array}{ll} 0 & ,x\le a \\ \frac{1}{2}+\frac{1}{2}\cos(\frac{x-b}{b-a}\pi) & ,a\le x\le b \\ 1 & ,b\le x\le c \\ \frac{1}{2}+\frac{1}{2}\cos(\frac{x-c}{d-c}\pi) & ,c\le x\le d \\ 0 & ,d\le x \end{array}\right\}, x\in\mathbb{R}$$



Formulation of Commonly Used MF

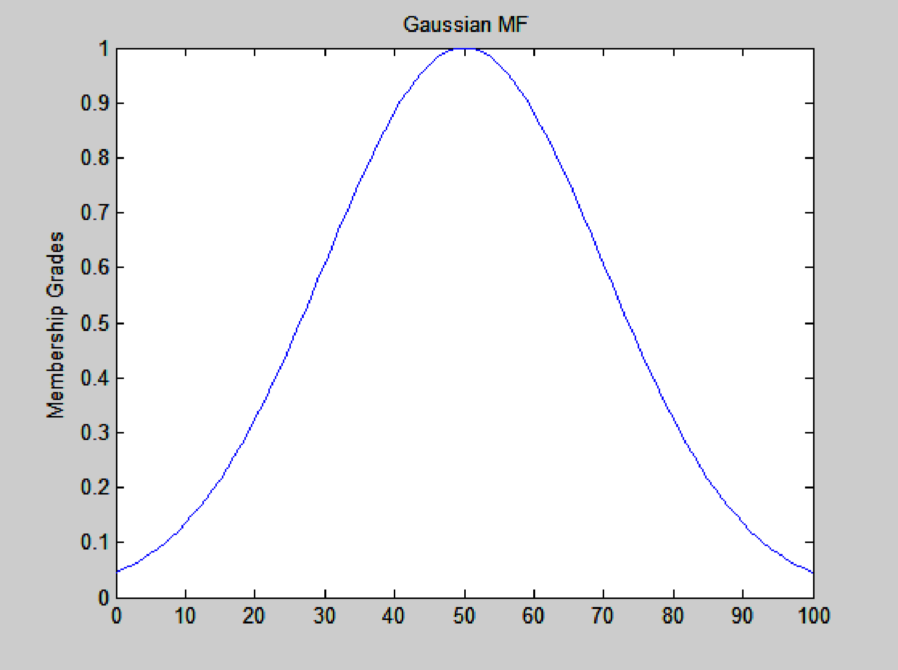
MFs of one dimension

- Gaussian MF: is specified by two parameters {c , σ } as follows:

$$gaussian(x,c,\sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

Matlab code

```
clear; clc;
x = 0:100;
mf =  gaussmf(x, [20 50]);
plot(x, mf);
%axis([-inf inf 0 1.2]);
ylabel('Membership Grades');
 title(' Gaussian MF');
```



Formulation of Commonly Used MF

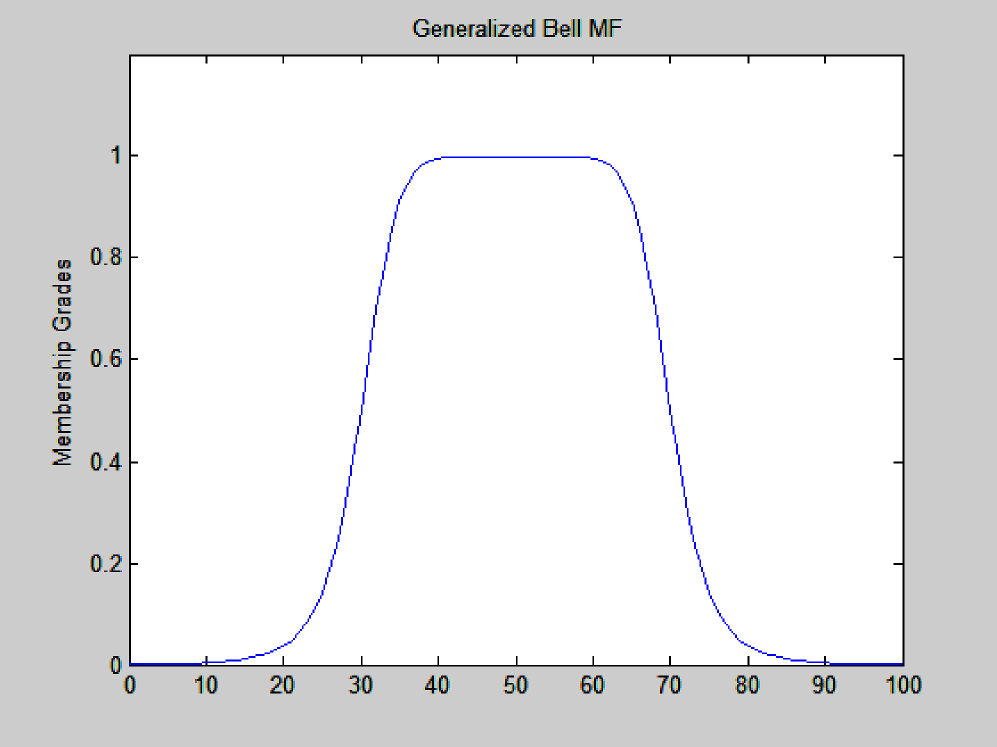
MFs of one dimension

- **Generalized bell MF**: is specified by three parameters {a, b, c } as follows:

$$bell(x,a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$

Matlab code

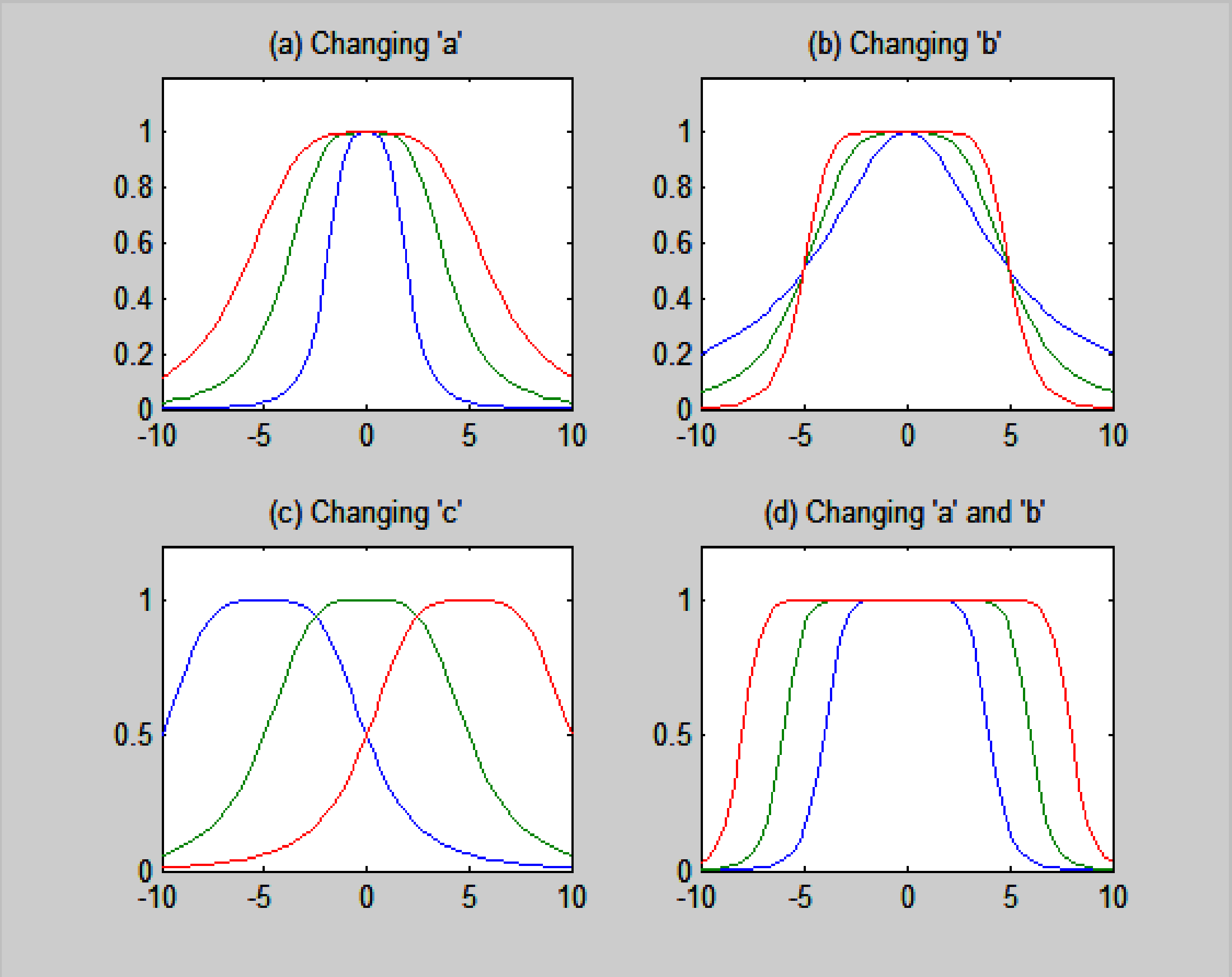
```
clear; clc;
x = 0:100;
mf = gbellmf (x, [20, 4, 50]);
 plot (x, mf);
axis([-inf inf 0 1.2]);
Ylabel ('Membership Grades');
 title(' Generalized Bell MF');
```



--------**Generalized bell MF**:

- Effects of the parameters

$$bell(x,a,b,c)=\frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$



Matlab code

```
x = (-10:0.4:10)';
b = 2;
c = 0;
mf1 = gbellmf(x, [2, b, c]);
mf2 = gbellmf(x, [4, b, c]);
mf3 = gbellmf(x, [6, b, c]);
mf = [mf1 mf2 mf3];
subplot(221); plot(x, mf); title('(a)
Changing ''a''');
axis([-inf inf 0 1.2]);

a = 5;
c = 0;
mf1 = gbellmf(x, [a, 1, c]);
mf2 = gbellmf(x, [a, 2, c]);
mf3 = gbellmf(x, [a, 4, c]);
mf = [mf1 mf2 mf3];
subplot(222); plot(x, mf); title('(b)
Changing ''b''');
axis([-inf inf 0 1.2]);
```

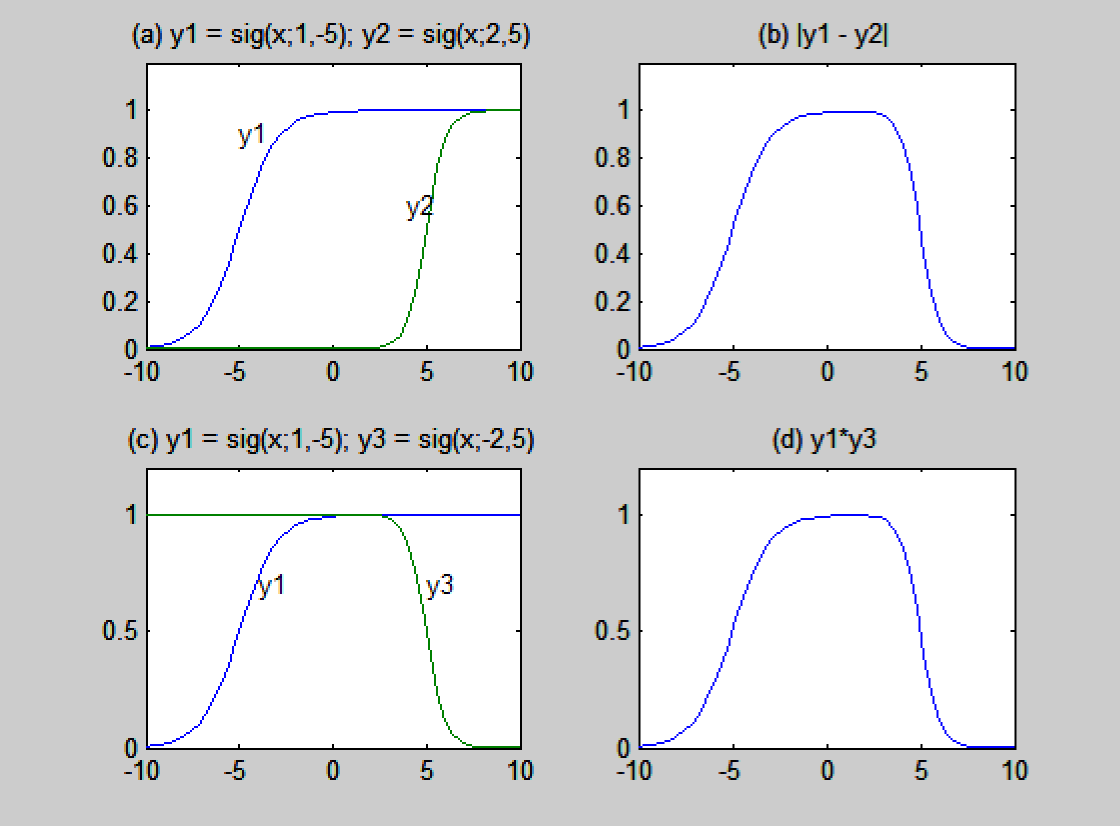
```
a = 5;
b = 2;
mf1 = gbellmf(x, [a, b, -5]);
mf2 = gbellmf(x, [a, b, 0]);
mf3 = gbellmf(x, [a, b, 5]);
mf = [mf1 mf2 mf3];
subplot(223); plot(x, mf); title('(c) Changing
''c''');
axis([-inf inf 0 1.2]);

c = 0;
mf1 = gbellmf(x, [4, 4, c]);
mf2 = gbellmf(x, [6, 6, c]);
mf3 = gbellmf(x, [8, 8, c]);
mf = [mf1 mf2 mf3];
subplot(224); plot(x, mf); title('(d) Changing
''a'' and ''b''');
axis([-inf inf 0 1.2]);
```

Commonly Used MFs

- **Sigmoidal MF**: **is defined by:**

$$sig(x,a,c) = \frac{1}{1+\exp[-a(x-c)]}$$

Where a controls the slope at x=c



Formulation of Commonly Used MF

MFs of one dimension

- **Sigmoidal MF**: Matlab code used for last fig:

```
x = (-10:0.4:10)';
y1 = sigmf (x, [1, -5]);
y2 = sigmf (x, [2, 5]);
y3 = sigmf (x, [-2, 5]);

subplot(221); plot (x, y1, x, y2);
text(-5, 0.9, 'y1'); text(4, 0.6, 'y2');
axis([-inf inf 0 1.2]);
title ('(a) y1 = sig(x;1,-5); y2 =
sig(x;2,5)');

subplot(222); plot(x, y1-y2);
axis([-inf inf 0 1.2]);
title ('(b) |y1 - y2|');
```

```
subplot(223); plot(x, y1, x, y3);
text(-4, 0.7, 'y1'); text(5, 0.7, 'y3');
axis([-inf inf 0 1.2]);
title ('(c) y1 = sig(x;1,-5); y3 = sig(x;-2,5)');

subplot(224); plot(x, y1.*y3);
axis([-inf inf 0 1.2]);
title ('(d) y1*y3');
```

Cylindrical Extension

MF of two dimensions

- MFs with two inputs (each from different UOD) is referred to as two-dimensional MFs
- One natural way to extend one-dimensional MFs to two-dimensional MFs is via ***cylindrical extension***, defined next:
- If A is a fuzzy set in X, then the cylindrical extension in the Cartesian space is a fuzzy set defined by:

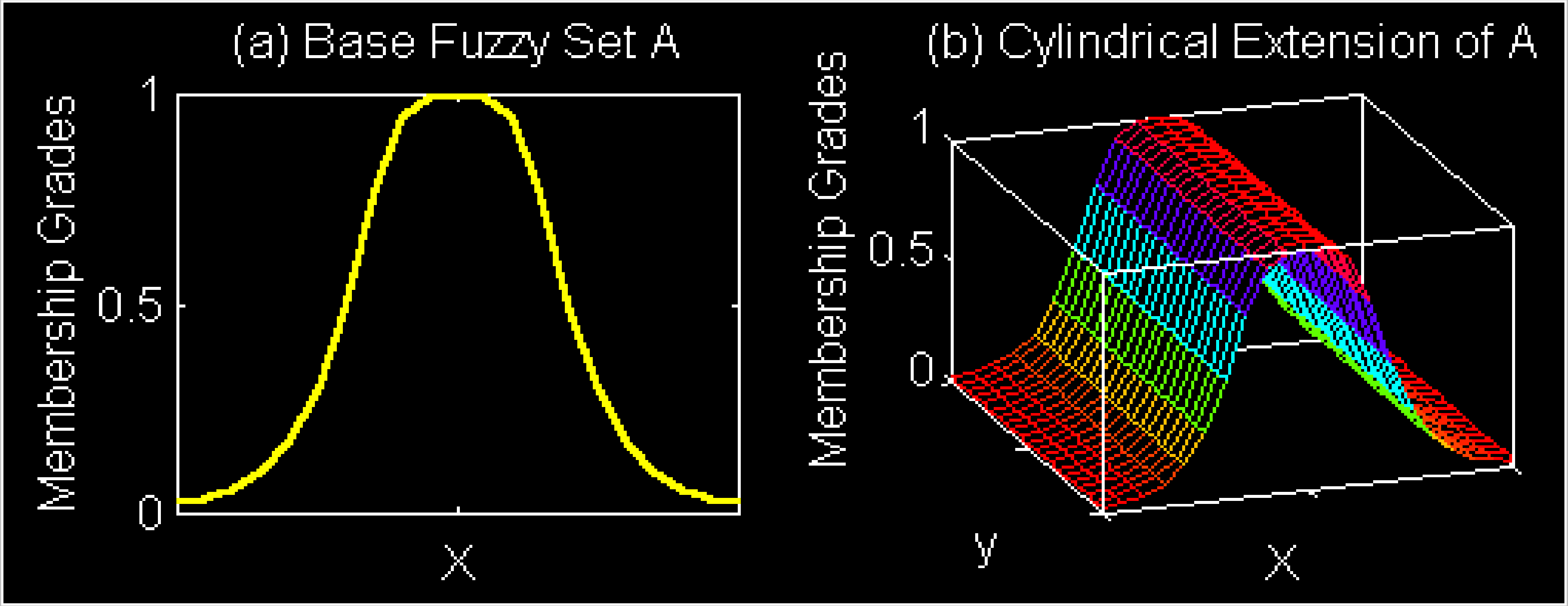
$$C(A) = \int_{X \times Y} \mu_A(x)/(x,y)$$

Cylindrical Extension

MF of two dimensions

Base set A

Cylindrical Ext. of A



cyl_ext.m